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On ΠG^B*- Closed Sets in Topological Spaces

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Abstract: In this Paper we introduce a new class of sets called π generalized ^ b*-closed set (briefly πg^b*-closed) and some of its characteristics are investigated. Further we studied the concepts of πg^b*-open sets and πg^b*-$T_{1/2}$ space.

Key words: πg^b*-closed sets, πg^b*-open sets, πg^b*-$T_{1/2}$ space, πg^b*-closure operator.

I. INTRODUCTION

Levine[11] and Andrijevic[3] introduced the concept of generalized open sets and b-open sets respectively in topological spaces. The class of b-open sets is contained in the class of semipre-open sets and contains the class of semi-open and the class of pre-open sets. Since then several researches were done and the notion of generalized semi-closed, generalized pre-closed and generalized semipre-open sets were investigated. In 1968 Zaitsev[19] defined π-closed sets. Later Dontchev and Noiri[8] introduced the notion of πg-closed sets. Park[15] defined πgp-closed sets. Then Aslim, Caksu and Noir[4] introduced the notion of πgs-closed sets. The idea of πgb-closed sets were introduced by D.Sreeja and C. Janaki[18]. Later the properities and characteristics of πgb-closed sets were introduced by Sinem Caglar and Gulhan Ashim[17].
The aim of this paper is to investigate the notion of πg^b*-closed sets and its properties. In section 3 we study the basic properties of πg^b*-closed sets. In section 4 some characteristics of πg^b*-closed sets are introduced and the idea of πg^b*-$T_{1/2}$ space is discussed.

II. PRELIMINARIES

Throughout this paper (X,τ) represents non empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. A subset A of a topological space (X,τ), cl(A) and int(A) denote the closure of A and interior of A respectively. (X,τ) will be replaced by X if there is no chance of confusion.
Definiton: Let (X,τ) be a topological space. A subset A of (X,τ) is called

(1) a semi-closed set if int(cl(A))⊆A.
(2) a α-closed set if cl(int(cl(A)))⊆A.
(3) a pre-closed set if cl(int(A))⊆A.
(4) a semipre-closed set if int(cl(int(A)))⊆A.
(5) a regular-closed set if A=cl(int(A)).
(6) a b-closed set if cl(int(A))∩int(cl(A))⊆A.
(7) a b*-closed set if int(cl(A))⊂U, whenever A⊂U and U is b-open.

the complements of the above mentioned sets are called semi-open, α-open, pre-open, semi-open, regular open, b-open, b*-open sets respectively. The intersection of all semi-closed (resp. α-closed, pre-closed, semipre-closed, regular-closed and b-closed) subsets of (X,τ) containing A is called the semi-closure (resp. α-closure, pre-closure, semipre-closure, regular-closure and b-closure) of A and is denoted by scl(A) (resp. αcl(A), pcl(A), spcl(A), rcl(A) and bcl(A)). A subset A of (X,τ) is called clopen if it is both open and closed in (X,τ).

Definition
A subset A of a space (X,τ) is called π-closed if A is finite intersection of regular closed sets.
Definition
A subset A of a space (X,τ) is called
(1) a g-closed set if cl(A)⊂U whenever A⊂U and U is open in (X,τ).
(2) a gp-closed set if pcl(A)⊂U whenever A⊂U and U is open in (X,τ).
(3) a gs-closed set if scl(A)⊂U whenever A⊂U and U is open in (X,τ).
(4) a gb-closed set if bcl(A)⊂U whenever A⊂U and U is open in (X,τ).
(5) a gα-closed set if αcl(A)⊂U whenever A⊂U and U is open in (X,τ).
(6) a πg-closed set if cl(A)⊂U whenever A⊂U and U is π-open in (X,τ).
(7) a πgα-closed set if αcl(A)⊂U whenever A⊂U and U is π-open in (X,τ).
(8) a πgp-closed set if pcl(A)⊂U whenever A⊂U and U is π-open in (X,τ).
(9) a πgs-closed set if scl(A)⊂U whenever A⊂U and U is π-open in (X,τ).
(10) a πgb-closed set if bcl(A)⊂U whenever A⊂U and U is π-open in (X,τ).

Complement of π-closed set is called π-open set.
Complement of g-closed, gp-closed, gs-closed, gb-closed, gα-closed, πgα-closed, πgp-closed, πgs-closed and πgb-closed sets are called g-open, gp-open, gs-open, gb-open, gα-open, πgα-open, πgp-open, πgs-open and πgb-open sets respectively.

Definition
Let (X,τ) be a topological space then a set A⊆(X,τ) is said to be Q-set if int(cl(A))=cl(int(A)).

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III. πg^b*-CLOSED SETS IN TOPOLOGICAL SPACES

Definition
A subset A of a space (X, τ) is called πg^b*-closed set if int(bcl(A))⊂U whenever A⊂U and U is πg-open in (X, τ).

Theorem: 3.1
Every g-closed set is πg^b*-closed.

Proof
Let A be a g-closed set of (x,τ) such that A⊆U and U is πg-open in X. Since cl(A)⊂U. As bcl(A) ⊂ cl(A) ⊂ U, int(bcl(A)) ⊆ int(U) = U. Hence A is πg^b*-closed.

Remark: 3.1
The converse of the above theorem is not true as seen from the following example.

Example: 3.1
Let X={a,b,c} and τ={X,Φ,{a},{b},{a,b}}. Let A={{a},{b}}. Then A is πg^b*-closed but not g-closed.

Theorem: 3.2
Every π-closed set is πg^b*-closed.

Proof
Let A be a π-closed set and A⊆U, U is πg-open. since πcl(A)=A, int(bcl(A))⊂πcl(A)=A, therefore int(bcl(A))⊂A whenever A⊂U and U is πg-open. Hence A is πg^b*-closed.

Remark: 3.2
The converse of the above theorem is not true as seen from the following example.

Example: 3.2
Let X={a,b,c,d} and τ={X,Φ,{a},{b},{a,b},{b,c},{a,b,c},{a,b,d}}. Let A={{a,c,d},{a,c}}. Then A is πg^b*-closed but not π-closed.

Theorem: 3.3
Every closed set is πg^b*-closed.

Proof
Let A be a closed set of (x,τ) such that A⊆U and U is πg-open in X. since bcl(A) ⊂ cl(A) = A, int(bcl(A)) ⊂ int(A) ⊆ int(U) = U. Hence A is πg^b*-closed.

Remark: 3.3
The converse of the above theorem is not true as seen from the following example.

Example: 3.3
Let X={a,b,c} and τ={X,Φ,{a},{b},{a,b}}. Let A={b}. Then A is πg^b*-closed but not closed.

Theorem: 3.4
Every α-closed set is πg^b*-closed.

Proof
Let A be a α-closed set of (x,τ) such that A⊆U and U is πg-open in X. Since bcl(A) ⊂ αcl(A) = A, int(bcl(A)) ⊂ int(A) ⊆ int(U) = U. Hence A is πg^b*-closed.

Remark: 3.4
The converse of the above theorem is not true as seen from the following example.

Example: 3.4
Let X={a,b,c} and τ={X,Φ,{a},{b},{a,b}}. Let A={a}. Then A is πg^b*-closed but not α-closed.

Theorem: 3.5
Every pre closed set is πg^b*-closed.

Proof
Let A be a pre closed set of (x,τ) such that A⊆U and U is πg-open in X. Since bcl(A) ⊂ pcl(A) = A, int(bcl(A)) ⊂ int(A) ⊆ int(U) = U. Hence A is πg^b*-closed.

Remark: 3.5
The converse of the above theorem is not true as seen from the following example.

Example: 3.5
Let X={a,b,c,d} and τ={X,Φ,{a},{d},{a,d},{c,d},{a,c,d}}. Let A={c,d}. Then A is πgb**-closed but not pre closed.

Theorem: 3.6
Every gb-closed set is πg^b*-closed.

Proof
Let A be a gb-closed set of (x,τ) such that A⊆U and U is πg-open in X. since every πg-open set is open. bcl(A) ⊂ U. Thus int(bcl(A)) ⊆ int(U) = U. Hence A is πg^b*-closed.

Remark: 3.6
The converse of the above theorem is not true as seen from the following example.

Example: 3.6
Let X={a,b,c,d} and τ={X,Φ,{b},{c,d},{b,c,d}}. Let A={b,d}. Then A is πgb**-closed but not α-closed.

Theorem: 3.7
Every πgα-closed set is πg^b*-closed.

Proof
Let A be a πgα-closed set of (x,τ) such that A⊆U and U is πg-open in X. Then αcl(A) ⊂ U, bcl(A)⊂αcl(A)⊂U, int(bcl(A)) ⊂ int(A) ⊆ int(U) = U. Hence A is πg^b*-closed.

Remark: 3.7
The converse of the above theorem is not true as seen from the following example.

Example: 3.7
Let X={a,b,c} and τ={X,Φ,{a},{b},{a,b},{a,c}}. Let A={a}. Then A is πg^b*-closed but not πgα-closed.

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Theorem: 3.8
Every πg^b*-closed set is πgb-closed.

Proof
Let A be a πg^b*-closed set of (x,τ) such that A⊆U and U is π-open in X. since A is πg^b*-closed set, bcl(A)⊆U and, hence bcl(A)⊆U. Then A is πgb-closed.

Remark: 3.8
The converse of the above theorem is not true as seen from the following example.

Example: 3.8
Let X={a,b,c,d} and τ={X,Φ,{a},{b},{a,b},{a,b,c},{a,b,d}}. Let A={a,b,c}. Then A is πgb-closed but not πg^b*-closed.

Theorem: 3.9
Every πg^b*-closed set is πgs-closed.

Proof
Let A be a πg^b*-closed set of (x,τ) such that A⊆U and U is π-open in X. since A is πg^b*-closed set, intbcl(A)⊆U and, hence bcl(A)⊆scl(A)⊆U, bcl(A)⊆U. Then A is πgs-closed.

Remark: 3.9
The converse of the above theorem is not true as seen from the following example.

Example: 3.9
Let X={a,b,c,d} and τ={X,Φ,{b},{b,c}}. Let A={a,b,d}. Then A is πgs-closed but not πgb**-closed.

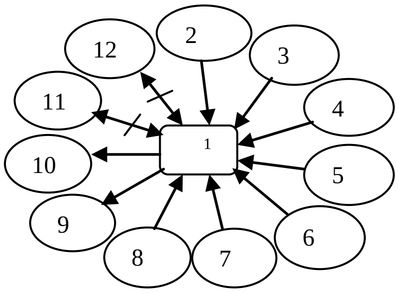
Remark: 3.10
The concept of πgp-closed set and πg^b*-closed set are independent of each other. It is shown in the following example.

Example: 3.10
Let X={a,b,c} and τ= {X, Φ, {a},{b},{a,b},{a,c}}. In this topological space the subset A={a,b} is πgp-closed but not πg^b*-closed set and the subset B={a} is πg^b*-closed but not πgp-closed set.

Remark: 3.11
The concept of πg-closed set and πg^b*-closed set are independent of each other. It is shown in the following example.

Example: 3.11
Let X={a,b,c,d} and τ= {X, Φ, {a},{b},{a,b},{b,c},{a,b,c},{a,b,d}}. In this topological space the subset A={a,d} is πg-closed but not πg^b*-closed set and the subset B={a} is πg^b*-closed but not πg-closed set.

The above discussions are summarized in the following diagram



(1) πg^b*-closed set, (2) g-closed set, (3) π-closed set, (4) closed set, (5) α-closed set, (6) pre-closed set, (7) gb-closed set, (8) πgα-closed set, (9) πgb-closed set, (10) πgs-closed set, (11) πgp-closed set, (12) πg-closed set.

IV. CHARACTERISTICS OF πg^b*-CLOSED SETS

Remark 4.1
Finite union of πg^b*-closed sets need not be πg^b*-closed which can be seen the following example.

Example 4.1
Let X={a,b,c} with topology τ={X,Φ,{a},{b},{a,b},{a,c}}. Let A={a} and B={b} then both A and B are πg^b*-closed. But, A⋃B={a,b} is not πg^b*-closed.

Remark 4.2
Finite intersection of πg^b*-closed sets need not be πg^b*-closed which can be seen the following example.

Example 4.2
Let X={a,b,c,d} with topology τ={X,Φ,{a},{b},{a,b},{a,b,c},{a,b,d}}. Let A={b,d} and B={b,c,d} then both A and B are πg^b*-closed. But, A⋂B={b,d} is not πg^b*-closed.

Theorem 4.1
Let (x,τ) be a topological space if A⊂X is πg^b*-closed set then int(bcl(A))-A does not contain any non empty πg-closed set.

Proof
Let A be a πg^b*-closed set in (X,τ) and F⊂ int(bcl(A))-A such that F is πg-closed in X. Then (X-F) is πg-open in X and A⊆(X-F). since A is πg^b*-closed, int(bcl(A))⊂(X-F)⇒F⊂(X-int(bcl(A))) therefore F⊂(int(bcl(A))-A)⋂(X-int(bcl(A)))⇒F=Φ. Therefore int(bcl(A))-A does not contain any non empty πg-closed set.

Theorem 4.2
If A is a πg^b*-closed and B is any set such that A⊆B⊆int(bcl(A)), then B is a πg^b*-closed.

Proof
Let B⊆U and U be πg-open. since A⊆B⊆U and A is πg^b*-closed, int(bcl(A))⊆U. Now int(bcl(B))⊆int(bcl(A))⊆U. Hence B is πg^b*-closed.

Theorem 4.3
Let (X,τ) be a topological space if A⊂X

Definition: 4.1
A set $A \subset X$ is called πg^$b*$-open if and only if its complement is πg^$b*$-closed in X.

Theorem 4.4
A subset $A \subseteq X$ is πg^$b*$-open if and only if $F \subseteq cl(bint(A))$ whenever F is πg-closed and $F \subseteq A$.

Proof
Assume that $A \subseteq X$ is πg^$b*$-open set in X. Let F be πg-closed such that $F \subseteq A$. Then $(X-A) \subset (X-F)$, since (X-A) is πg^$b*$-closed and (X-F) is πg-open, $int(bcl(X-A)) \subseteq (X-F) \Rightarrow (X-cl(bcl(A))) \subseteq (X-F)$. Hence $F \subseteq cl(bcl(A))$. Conversely, assume that F is πg-closed and $F \subseteq A$ such that $F \subseteq cl(bcl(A))$. Let $(X-A) \subseteq U$, where U is πg-open. Then $(X-U) \subseteq A$ and since $(X-U) \subseteq cl(bcl(A)) \Rightarrow int(bcl(X-A)) \subseteq U$. Hence (X-A) is πg^$b*$-closed and A is πg^$b*$-open.

Theorem 4.5
If $cl(bint(A)) \subseteq B \subseteq A$ and A is πg^$b*$-open, then B is πg^$b*$-open.
Proof
Let F be a πg-closed set such that $F \subseteq B$. Since $B \subseteq A$ we get $F \subseteq A$. Given A is πg^$b*$- open thus $F \subseteq cl(bint(A)) \subseteq cl(bint(B))$. Therefore B is πg^$b*$-open.

Definition 4.2
A space (X,τ) is called a πg^$b*$-$T_{1/2}$ space if every πg^$b*$-closed set is b*-closed.

Theorem 4.6
For a topological space (X,τ) the following are equivalent
1) X is πg^$b*$-$T_{1/2}$
2) $\forall$ subset $A \subseteq X$, A is πg^$b*$-open if and only if A is b*-open.
Proof
$(1) \Rightarrow (2)$
Let $A \subseteq X$ be πg^$b*$-open. Then (X-A) is πg^$b*$-closed and by (1). (X-A) is b*-closed $\Rightarrow$ A is b*-open. conversely assume A is b*-open. Then (X-A) is b*-closed. As every b*-closed set is πg^$b*$-closed, (X-A) is πg^$b*$-closed $\Rightarrow$ A is πg^$b*$-open. $(2) \Rightarrow (1)$
Let A be a πg^$b*$-closed set in X. Then (X-A) is πg^$b*$-open. Hence by (2) (X-A) is b*-open $\Rightarrow$ A is b*-closed. Hence X is πg^$b*$-$T_{1/2}$.

Theorem 4.7
Let (X,τ) be a πg^$b*$-$T_{1/2}$ space then every singleton set is either πg-closed or b*-open.
Proof
Let $x \epsilon X$ suppose $\{x\}$ is not πg-closed. Then X-{x} is not πg-open. Hence X-{x} is trivially πg^$b*$-closed. Since X is πg^$b*$-$T_{1/2}$ space, X-{x} is b*-closed$\Rightarrow$\{x\} is b*-open.

Definition 4.3
The intersection of all πg^$b*$-closed set containing A is called the πg^$b*$-closure of A denoted by πg^$b*$-cl(A).

Theorem 4.8
Let $A \subseteq (X,\tau)$ and $x \epsilon X$. Then $x \epsilon$ πg^$b*$-cl(A) if and only if $V \cap A \neq \Phi$ for every πg^$b*$-open set V containing x.
Proof
Suppose $x \epsilon$ πg^$b*$-cl(A) and let V be an πg^$b*$-open set such that $x \epsilon V$. Assume $V \cap A = \Phi$, then $A \subset X/V \Rightarrow \pi gb**$-cl(A) $\subset X/V \Rightarrow x \epsilon X/V$, a contradiction. Thus $V \cap A \neq \Phi$ for every πg^$b*$-open set V containing x. To prove the converse suppose $x \notin \pi g$^$b*$-cl(A) $\Rightarrow x \in X/ \pi g$^$b*$-cl(A)=V (say). Then V is a πg^$b*$-open and $x \in V$. Also since $A \subseteq \pi g$^$b*$-cl(A) $\Rightarrow A \not\subset V \Rightarrow V \cap A = \Phi$. Hence the theorem.

Theorem 4.9
For set $A \subseteq (X,\tau)$ if A is πg-clopen then A is πg-open, Q-set, πg^$b*$-closed set.
Proof
Let A be πg-clopen. Then A is both πg-open and πg-closed. Hence A is both open and closed. Therefore, cl(int(A))=int(cl(A)), thus A is a Q-set. As $bcl(A) \subseteq cl(A)$-A. $int(bcl(A)) \subseteq int(A)=A$.

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